Online Appendix to
Inflation analysis with semi-structural models

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Abstract

This online appendix provides details on the dataset and the estimation of the models described in 'Inflation analysis with semi-structural models', as well as additional results and robustness exercises.

Keywords: real-time forecasting, output gap, Phillips curve, semi-structural models, Bayesian estimation.
JEL Classification: C11, C32, C53, E31, E32, E52.

Appendix A Data

A.1 GDP SPF

The Survey of Professional Forecasters includes expectations for real GDP in levels and growth rates. We decided not to use the official release for the expectation of real GDP in levels, because it is not adjusted for changes in the basis year, data revisions and in the seasonal adjustment mechanism.

Instead, we computed the one-year ahead SPF expectation for the growth rates and we used it jointly with the latest vintage of data available for real GDP to compute an adjusted prediction for the levels.

Appendix B Stylised model – output gap and RE

In this appendix we show that a version of the quarterly model without idiosyncratic trends is compatible with the solution of a New Keynesian model with rational expectations.[1]

In the quarterly model, we assume that

$$\pi_t = \tau_t^{\pi} + \gamma_{\pi,0}\psi_t^{gap} + \gamma_{\pi,1}\psi_{t-1}^{gap} + \delta_{\pi,0}\psi_t^{epc} + \delta_{\pi,1}\psi_{t-1}^{epc} \ , \tag{1}$$

while the output gap and the energy price cycles are AR(2) process, that we can write as:

$$\psi_t^{gap} \quad = \quad \rho_1^{gap}\psi_{t-1}^{gap} + \rho_2^{gap}\psi_{t-2}^{gap} + \upsilon_t^{gap}, \tag{2}$$

$$\psi_t^{epc} \quad = \quad \rho_1^{epc}\psi_{t-1}^{epc} + \rho_2^{epc}\psi_{t-2}^{epc} + \upsilon_t^{epc}. \tag{3}$$

Rational expectations agents would form model-consistent expectations about inflation as

$$\begin{aligned} E_t\pi_{t+1} &= E_t\left[\tau_{t+1}^{\pi} + \gamma_{\pi,0}\psi_{t+1}^{gap} + \gamma_{\pi,1}\psi_t^{gap} + \delta_{\pi,0}\psi_{t+1}^{epc} + \delta_{\pi,1}\psi_t^{epc}\right] \\ &= \tau_t^{\pi} + \gamma_{\pi,0}(\rho_1^{gap}\psi_t^{gap} + \rho_2^{gap}\psi_{t-1}^{gap}) + \gamma_{\pi,1}\psi_t^{gap} + \delta_{\pi,0}(\rho_1^{eps}\psi_t^{eps} + \rho_2^{eps}\psi_{t-1}^{eps}) + \delta_{\pi,1}\psi_t^{eps} \\ &= \tau_t^{\pi} + \gamma_{exp,1}\psi_t^{gap} + \gamma_{exp,2}\psi_{t-1}^{gap} + \delta_{exp,1}\psi_t^{epc} + \delta_{exp,2}\psi_{t-1}^{epc} \ . \end{aligned}$$

This shows that if the output gap is an AR(2) process, then rational expectations are a moving average of output gap and its first lag.

[1] See **?** for additional details on this point.

Let now consider a New Keynesian Phillips Curve connecting the cyclical components of output, inflation, and inflation expectations, of the form

$$\widehat{\pi}_t^{gap} = \beta \mathbb{E}_t \left[\widehat{\pi}_{t+1}^{gap} \right] + \kappa \widehat{y}_t^{gap} \ , \tag{4}$$

where hats indicate deviations from trends and ε_t is an i.i.d. white noise disturbance.

Substituting in the PC equation, one obtains:

$$\begin{aligned} \widehat{\pi}_t^{gap} &= \beta(\gamma_{exp,1}\psi_t^{gap} + \gamma_{exp,2}\psi_{t-1}^{gap} + \delta_{exp,1}\psi_t^{epc} + \delta_{exp,2}\psi_{t-1}^{epc}) + \kappa\psi_t^{gap} + \varepsilon_t \ , \\ &= (\beta\gamma_{exp,1} + \kappa)\psi_t^{gap} + \beta\gamma_{exp,2}\psi_{t-1}^{gap} + \beta\delta_{exp,1}\psi_t^{epc} + \beta\delta_{exp,2}\psi_{t-1}^{eps} \ , \end{aligned}$$

which is compatible with the assumed equilibrium dynamic of $\widehat{\pi}_t^{gap}$.

Appendix C Adaptive Metropolis-Within-Gibbs

C.1 Algorithm

The estimation algorithm is an improved version of the Metropolis-Within-Gibbs in **?** that employs the Single Component Adaptive Metropolis proposed in **?**.

This hybrid algorithm is structured in two blocks: (1) a Single Component Adaptive Metropolis (**?**) step for the estimation of the state-space parameters, (2) a Gibbs sampler (**??**) to draw the unobserved states conditional on the model parameters. Since we have non-stationary unobserved states, we use the Kalman filter with exact diffuse initial conditions (**??**) to compute the log-likelihood of the model. Finally, we used the priors in **?**.

Algorithm: Adaptive Metropolis-Within-Gibbs

Initialisation

Let $\mathscr{K} := \{1, \ldots, n_k\}$ and denote as $\mathbf{P}(\mathscr{K})$ a function that returns a random permutation of $\mathscr{K}$ (uniformly taken from the full set of permutations of $\mathscr{K}$). Let also $\boldsymbol{\theta}_0$ be a n_k dimensional vector corresponding to the initial value for the Metropolis parameters. This vector is associated to a high posterior mass.

Single component adaptive metropolis

let $m = 1$

for $j = 1, \ldots, 10000$

let $\mathbf{S}_j = \mathbf{P}(\mathscr{K})$

for each k in $\mathbf{S}_j$

1. Adaptation: Update the standard deviation of the proposal distribution

$$\sigma_{k,j} = \begin{cases} 1 & \text{if } j \leq 10, \\ \exp\Big(\alpha_{k,j-1} - 0.44\Big)\sigma_{k,j-1} & \text{otherwise,} \end{cases}$$

where $\alpha_{k,j-1}$ is the acceptance rate for the iteration $j-1$, for the parameter at position $S_{k,j}$. Besides, 44% is the standard target acceptance rate for single component Metropolis algorithms.

2. New candidate: Generate a candidate vector of parameters $\boldsymbol{\theta}_m^*$ such that

$$\theta_{l,m}^* = \begin{cases} \theta_{l,m-1} & \text{if } l \neq k, \\ \underline{\theta} \overset{iid}{\sim} \mathcal{N}\Big(\theta_{l,m-1}, \sigma_{k,j}\Big) & \text{otherwise,} \end{cases}$$

for $l = 1, \ldots, n_k$.

3. Accept-reject: Set

$$\boldsymbol{\theta}_m = \begin{cases} \boldsymbol{\theta}_m^* & \text{accept with probability } \eta_m, \\ \boldsymbol{\theta}_{m-1} & \text{reject with probability } 1 - \eta_m, \end{cases}$$

where

$$\eta_m := \min\left(1, \frac{p\Big[\mathbf{Y} \mid \mathbf{f}(\boldsymbol{\theta}_m^*)^{-1}\Big]\, p\Big[\mathbf{f}(\boldsymbol{\theta}_m^*)^{-1}\Big]\, J(\boldsymbol{\theta}_m^*)}{p\Big[\mathbf{Y} \mid f(\boldsymbol{\theta}_{m-1})^{-1}\Big]\, p\Big[\mathbf{f}(\boldsymbol{\theta}_{m-1})^{-1}\Big]\, J\Big[\boldsymbol{\theta}_{m-1}\Big]}\right),$$

$\mathbf{f}$ and J are defined below.

4. Increase counter: Increase m by one.

Gibbs sampling

For $j > 5000$ (burn-in period), use the univariate approach for multivariate time series of **?** to the simulation smoother proposed in **?** to sample the unobserved states, conditional on the parameters. In doing so, we follow the refinement proposed in **?**.

Burn-in period

Discard the output of the first $j = 1, \dots, 5000$ iterations.

Jacobian

As in **?** most parameters are bounded in their support (e.g. the variance parameters must be larger than zero). In order to deal with this complexity, this manuscript transforms the bounded parameters ($\boldsymbol{\Theta}$) so that the support of the transformed parameters ($\boldsymbol{\theta}$) is unbounded. Indeed, the Adaptive Metropolis-Within-Gibbs draws the model parameters in the unbounded space. At a generic iteration j, the following transformations have been applied to a generic parameter i with a Normal, Inverse-Gamma or Uniform prior:

$$
\begin{aligned}
\theta^N_{i,j} &= \Theta^N_{i,j} \\
\theta^{IG}_{i,j} &= \ln(\Theta^{IG}_{i,j} - a_i) \\
\theta^U_{i,j} &= \ln\left(\frac{\Theta^U_{i,j} - a_i}{b_i - \Theta^U_{i,j}}\right),
\end{aligned}
$$

where a_i and b_i are the lower and the upper bounds for the i-th parameter. These transformations are functions $f(\Theta) = \theta$, with inverses $f(\theta)^{-1} = \Theta$ given by:

$$
\begin{aligned}
\Theta^N_{i,j} &= \theta^N_{i,j} \\
\Theta^{IG}_{i,j} &= \exp(\theta^{IG}_{i,j}) + a_i \\
\Theta^U_{i,j} &= \frac{a_i + b_i \exp(\theta^U_{i,j})}{1 + \exp(\theta^U_{i,j})}.
\end{aligned}
$$

These transformations must be taken into account when evaluating the natural logarithm of the prior densities by adding the Jacobians of the transformations of the variables:

$$\ln\left(\frac{d\Theta_{i,j}^{N}}{d\theta_{i,j}^{N}}\right) = 0$$

$$\ln\left(\frac{d\Theta_{i,j}^{IG}}{d\theta_{i,j}^{IG}}\right) = \theta_{i,j}^{IG}$$

$$\ln\left(\frac{d\Theta_{i,j}^{U}}{d\theta_{i,j}^{U}}\right) = \ln(b_i - a_i) + \theta_{i,j}^{U} - 2\ln(1 + \exp(\theta_{i,j}^{U})).$$

Appendix D Additional Quarterly Results

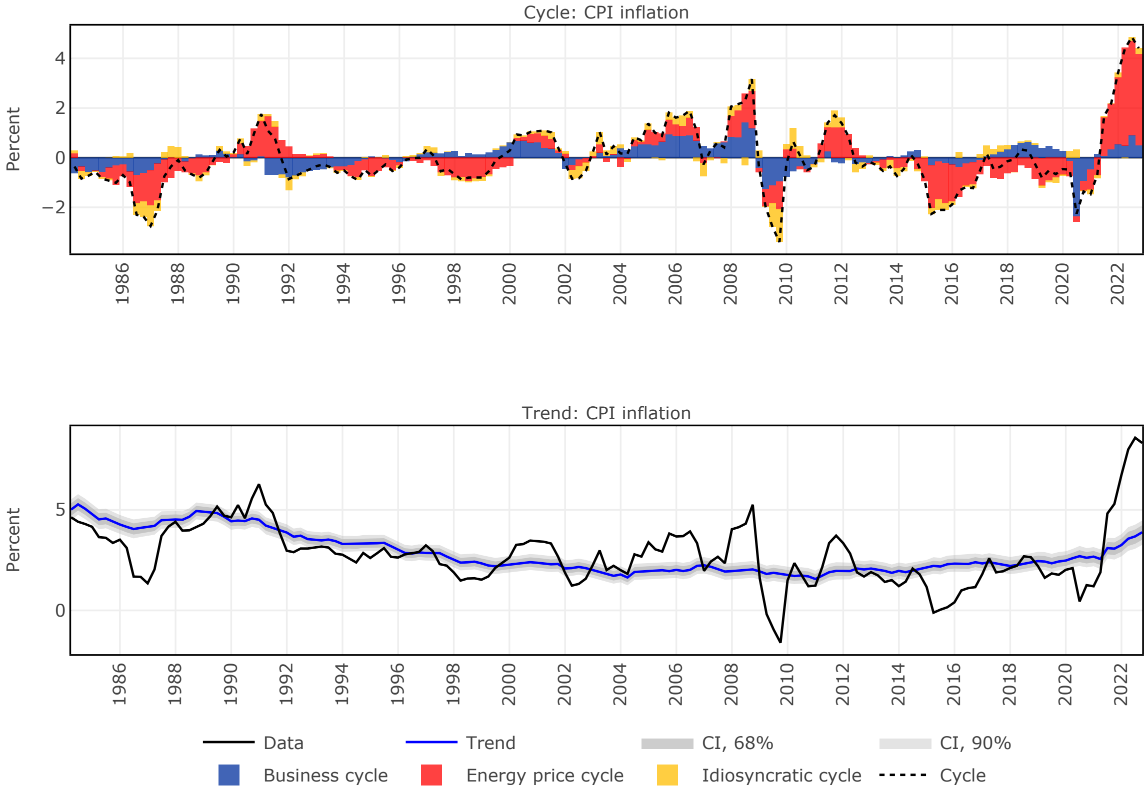

Figure 1: Top: Decomposition of the cycle of CPI inflation into common (in blue and red) and independent (in yellow) components, as estimated by the model in **?**. Bottom: Trend of CPI inflation (in blue), with relative coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model in **?**.

Appendix E Additional Real-Time Results

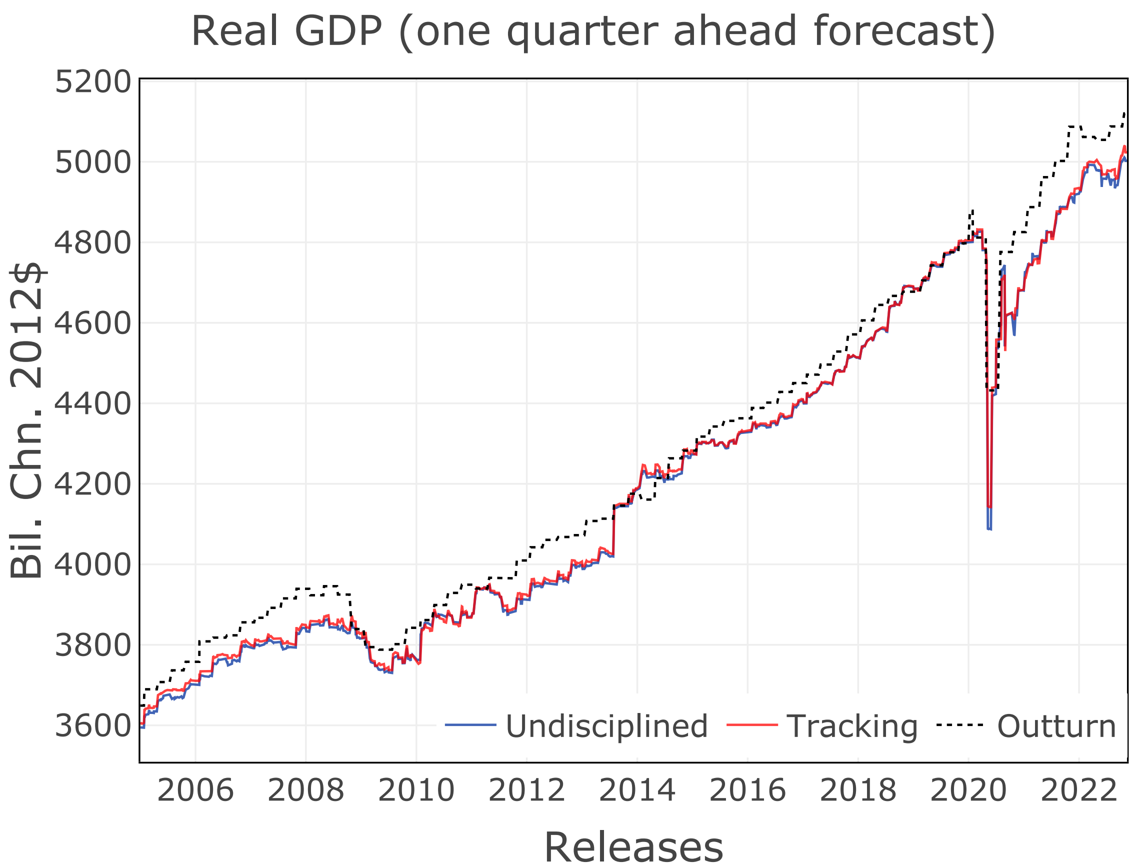

Figure 2: The chart reports the one quarter ahead, real time forecasts of Real GDP from the two models and compares them to the outturn. The out-of-sample evaluation starts in Jan-2005 and ends in Sept-2020.

Table 1: The first two rows of this table report the standard deviation of the output gap and potential output computed across vintages for each reference month and then averaged across reference months. The last two columns report the maximum absolute value of revisions computed for each reference month and then averaged across reference months.

	Output Gap		Potential Output	
	Undisciplined	Tracking	Undisciplined	Tracking
Mean of std dev	0.54	0.61	6.79	8.33
Mean of std dev (until 2005)	0.5	0.5	5.06	7.28
Mean of max revision	0.91	1.37	16.38	15.09
Mean of max revision (until 2005)	0.46	1.13	9.10	10.93

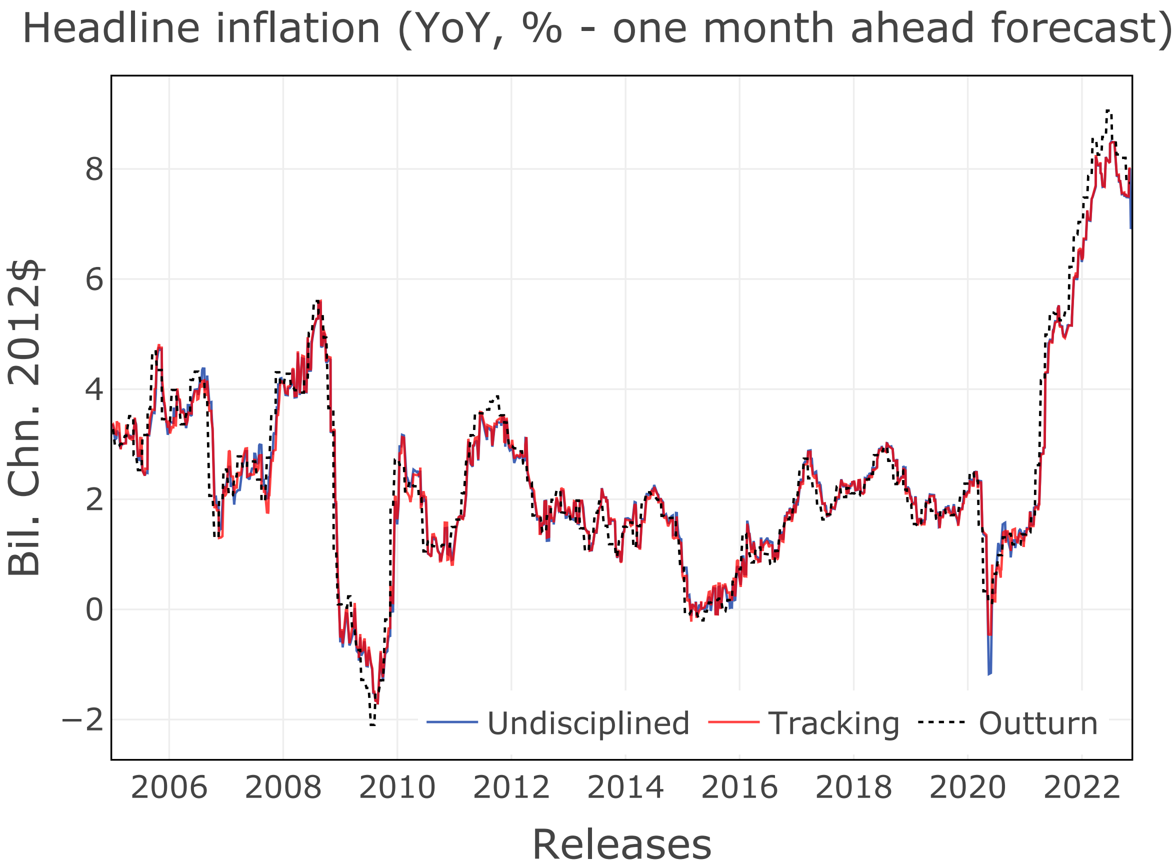

Figure 3: The chart reports the one month ahead, real time forecasts of inflation from the two models and compares them to the outturn. The out-of-sample evaluation starts in Jan-2005 and ends in Sept-2020.

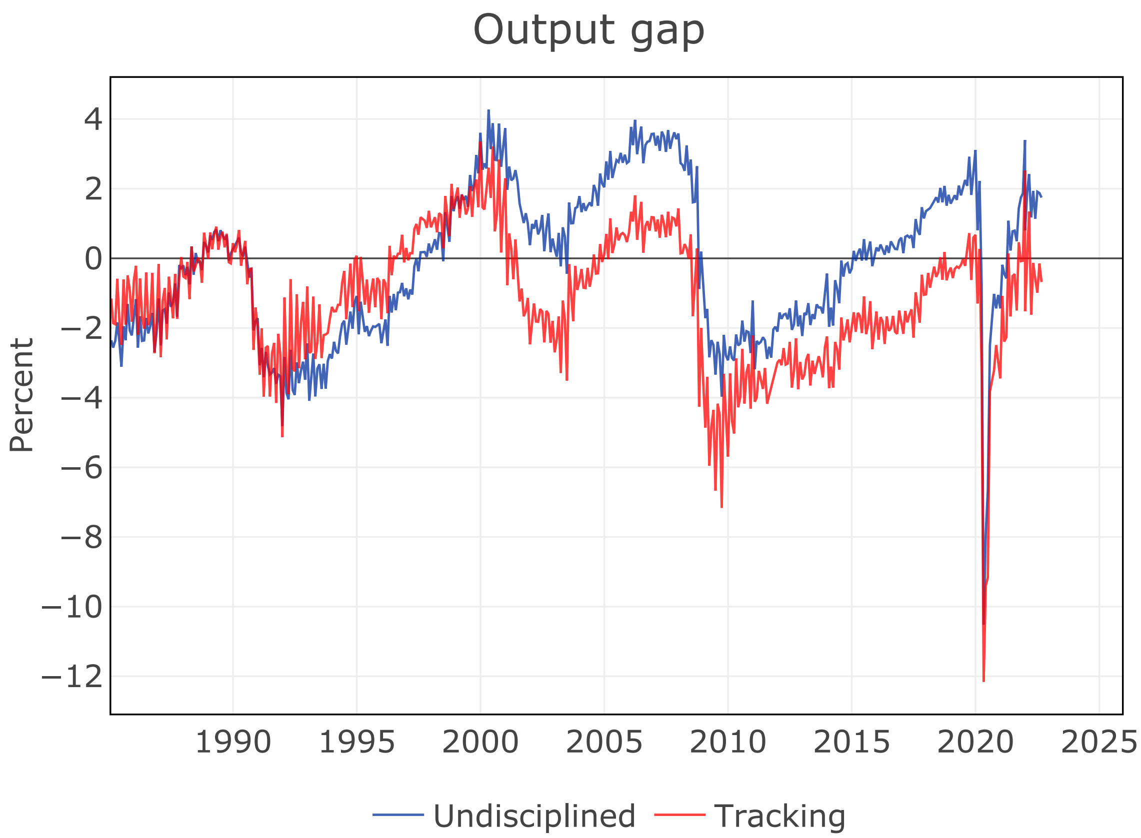

Figure 4: The chart compares the output gap estimates from the two models computed using the final (09/30/2020) data vintage from the out-of-sample forecasting exercise.